FARADAY AND LENZ

- Faraday's 1st law
  - "an electromotive force will be induced in a wire placed in a varying magnetic field."
- Faraday's 2nd law
  - "the magnitude of the induced EMF is proportional to the rate of change of the magnetic field in which the wire is placed."
- => $|e| \propto \frac{\partial B}{\partial t}$
- Lenz's law
  - "the current induced in a circuit due to a change in a magnetic field is directed to oppose the change in flux and to exert a mechanical force which opposes the motion."
- Modern version
  - $e = -\frac{d\phi}{dt}$
  - Recall that the negative sign means the direction of emf will induce a current to oppose the change in flux.

PHYSICS VS LCA (1)

- LCA: property of inductor
  - $V_L = L\frac{di_L}{dt}$ where V_L and i_L are voltage across and current flowing through the inductor, both defined in a passive convention.

- Physics: coil / inductor
  - Current i induces magnetic field B.
    - $B = \mu_0\, n\, i/l$ where n is a number of turns.
    - $\psi_m = B\ A$ where A is a cross-sectional area, $A = \pi r^2$ where r is a radius of the coil.
    - Therefore, $\psi_m = \mu_0 n\ i\ \pi r^2/l$.
    - Take derivative, $\frac{d\psi_m}{dt} = \mu_0 n\ \pi r^2\frac{di}{dt}$.
    - Since change in flux induces emf, then $e = -n\frac{d\ \psi_m}{d\ t} = -\frac{n^2\mu_0\pi r^2}{l}\frac{di}{dt} = -L\ \frac{di}{dt}$.
- ! What's going on here! LCA: $V_L = L\frac{di_L}{dt}$ vs. Physics: $emf = -L\frac{di_L}{dt}$.

PHYSICS VS LCA (II)

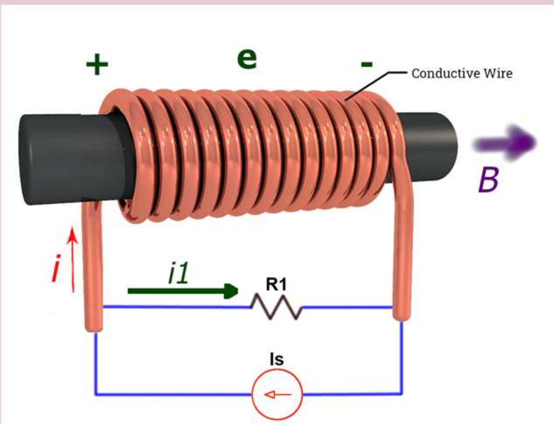
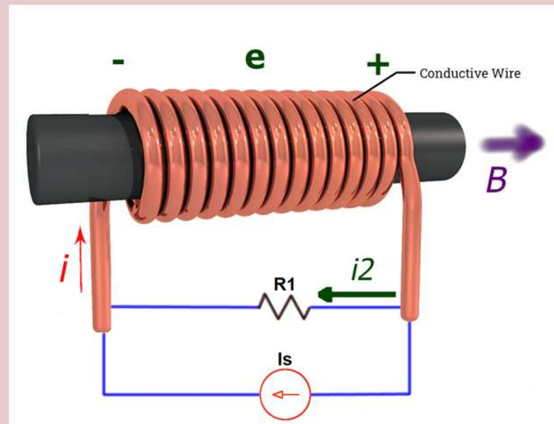
- Questions
  - 1. LCA defines V_L and i_L per passive convention. What are the direction of emf and i defined in physics?
  - 2. Recall that Faraday's is about magnitude of emf, but Lenz says the direction of emf is to induce the current to oppose the change in flux.
  - 3. Just additional notion: $n^2\mu_0\pi r^2 \equiv L$ is only valid for air core, if the inductor is made using other type of core the permeability must be change accordingly.

PHYSICS VS LCA (III)

Let's get into detail.

Case 1

- $\Delta i_s > 0 \rightarrow \Delta i > 0 \rightarrow \Delta B > 0 \rightarrow \Delta\Psi > 0 \rightarrow$ emf (in the direction inducing a current to oppose $\Delta\Psi$)

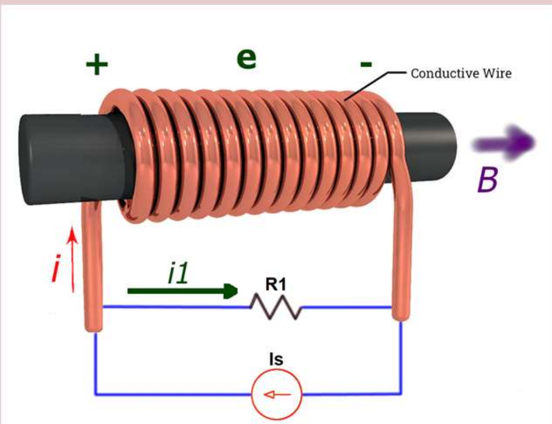
Scenario 1	**Scenario 2**
	
Here, $e > 0$ makes $i_1 > 0$. $i_1 > 0$ makes i decrease ($i = i_s - i_1$, from KCL). This **opposes** flux increase.	Here, $e > 0$ makes $i_2 > 0$. $i_2 > 0$ makes i increase ($i = i_s + i_2$, from KCL). This supports flux increase.

PHYSICS VS LCA (IV)

Let's get into detail.

Case II

- $\Delta i_s < 0 \rightarrow \Delta i < 0 \rightarrow \Delta B < 0 \rightarrow \Delta\Psi < 0 \rightarrow$ emf (in the direction inducing a current to oppose $\Delta\Psi$)

<table>
<tr><th>Scenario 1</th><th>Scenario 2</th></tr>
<tr><td>


Here, e > 0 makes $i_1 > 0$.
$i_1 > 0$ makes i decrease ($i = i_s - i_1$, from KCL).
This supports flux decrease.
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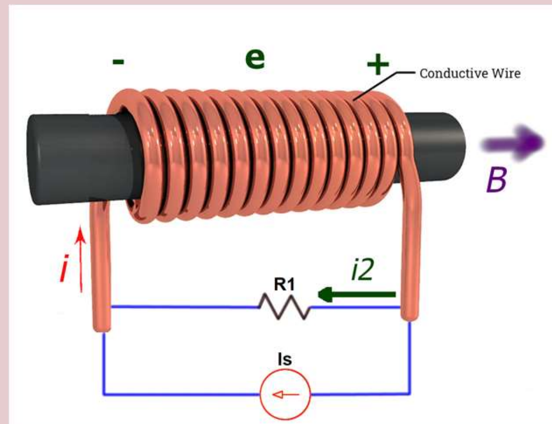

Here, e > 0 makes $i_2 > 0$.
$i_2 > 0$ makes i increase ($i = i_s + i_2$, from KCL).
This **opposes** flux decrease.
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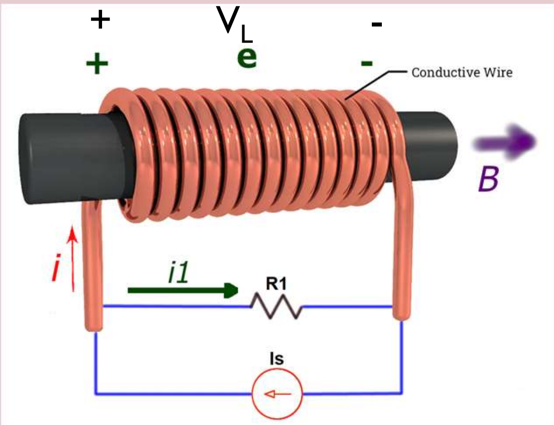
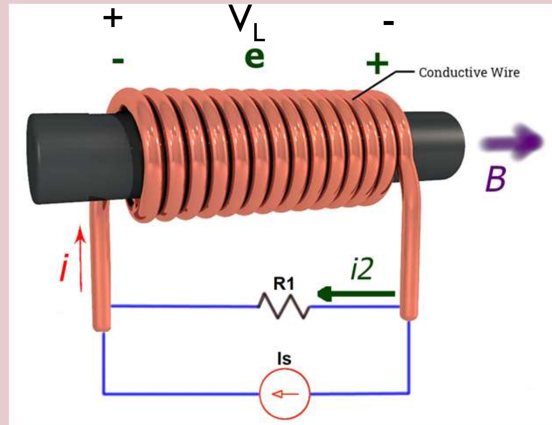
PHYSICS VS LCA (V)

Case 1: scenario 1	Case 2: scenario 2
	
$\Delta i_s > 0$ This **opposes** flux increase. Given passive con., $V_L > 0$ when di/dt > 0.	$\Delta i_s < 0$ This **opposes** flux decrease. Given passive con., $V_L < 0$ when di/dt < 0.

Verdict: in both cases, V_L has the same sign as di/dt.

That is, $V_L = L\frac{di}{dt}$, given the passive convention.

PHYSICS VS LCA (VI)

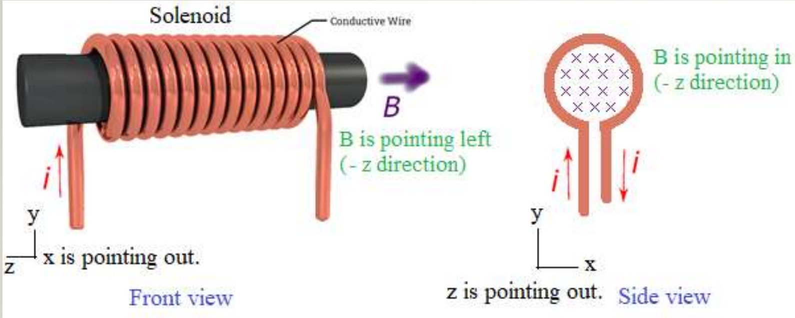
Let's do it properly with direction.

Recall $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ or $\oint_c \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt}\int_S \vec{B} \cdot \hat{n} da$

and $\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$ or $\oint_C \vec{B} \cdot d\vec{\ell} = \mu \left(I_{enc} + \varepsilon \frac{d}{dt}\int_S \vec{E} \cdot \hat{n}\, da\right)$.

Current *i* induces magnetic field .

Solenoid: $\vec{B} = \frac{\mu\, n\, i}{l}\,(-\hat{z})$; B is pointing to the $-\hat{z}$ direction; magnitude $|B| = \frac{\mu\, n\, i}{l}$.



$\vec{\psi} = \vec{B}\, A$ where A is a cross-sectional area, $A = \pi r^2$ where *r* is a radius of the coil.

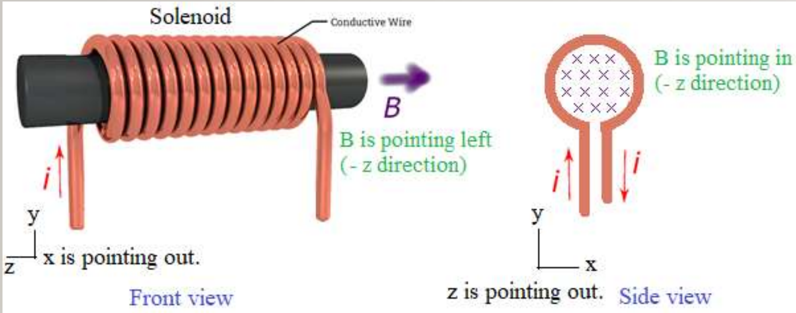
Therefore, $\vec{\psi} = \frac{\mu\, n\, i\, \pi r^2}{l}\,(-\hat{z})$ and

then take derivative, $\frac{d\vec{\psi}}{dt} = \frac{\mu\, n\, \pi r^2}{l}\frac{di}{dt}\,(-\hat{z})$.

Denote $L = \frac{\mu\, n\, \pi r^2}{l}$. Since change in flux induces emf, then

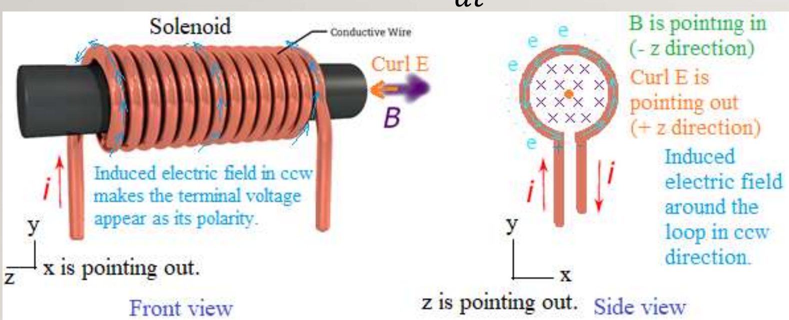
$e = \oint_c \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt}\int \vec{B} \cdot \hat{n} da = -\frac{d\vec{\psi}}{dt} = -L\frac{di}{dt}\,(-\hat{z})$.

PHYSICS VS LCA (VII)



$$e = \oint_c \vec{E} \cdot d\vec{\ell} = -L\frac{di}{dt}(-\hat{z}) = L\frac{di}{dt}\hat{z}.$$

If current i is increasing, i.e., $\frac{di}{dt} > 0$, then



Notice that when change is positive the induced voltage appeared at the terminal is positive, per **passive convention**.

Students are encouraged to work on other scenarios, e.g.,

- Current i is decreasing, $\frac{di}{dt} < 0$.
- Current is increasing, but coil is wound ccw, $\vec{B} = \frac{\mu\, n\, i}{l}\hat{z}$.
- Current is decreasing with coil wound ccw, $\vec{B} = \frac{\mu\, n\, i}{l}\hat{z}$.
- Current is flowing in another direction.
- …

- Note
  - if change is +, v_L (as passive convention) is +
  - and when change is -, v_L is –
  - then it approves passive convention.

RESIST TO CHANGE

- Lenz
  - "the current induced in a circuit due to a change in a magnetic field is directed to oppose the change in flux and to exert a mechanical force which opposes the motion."
- Le Chatelier
  - "When a simple system in thermodynamic equilibrium is subjected to a change in concentration, temperature, volume, or pressure, the system changes to a new equilibrium, and this change partly counteracts the applied change."
- Homeostasis
  - Homeostasis is brought about by a natural resistance to change when already in optimal conditions, and equilibrium is maintained by many regulatory mechanisms: it is thought to be the central motivation for all organic actions.